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Extended Chatterjea's Fixed Point Theorem on CM-Spaces Depended on Another Function

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Abstract: In this paper, we obtain a unique fixed point result that is ,extended Chattejea's fixed point theorem on CM-Spaces depended an another function . Our result is generalization results of some of the well known existing results in this literature.

Keywords: *Fixed point, contractive mapping, sequentially convergent and sub sequentially convergent.*

***2000 Subject Classification**: 47H10; 54H25.*

1. INTRODUCTION

In a Banach contraction mapping principle fixed point theorem is most cited one(see [4] or [6]), which asserts that if (X, ρ) is a CM(Complete Metric)- space and A: X→X is contractive mapping , that is there exists α϶[0,1) such that for all x, y ϵX ,ρ(Ax, Ay) ≤ α ρ(x, y). Then A has a unique fixed point. In 1968 Kannan [5] established a fixed point theorem which is also most cited result. Subsequently many authors studied and extended and generalized these results (see for e.g. [1-3], [7-11]). In 1977, B.E.Rhodes [11] considered different types of contractive conditions and canalized the relationship among them. In 2000 Branciari [4] introduced a class of generalized metric space. In 2008 Azam and Arshad [1] extended the Kannan's fixed point theorem in this kind of generalized metric spaces. Recently ,Moradi [7] obtained a result . In this paper we obtain a result, extended Chattejea's fixed point theorem on on CM-space which depends on another function.

2. PRELIMINARIES

The following are we needed getting the main results which ar due to [7].

Definition 1.1. Let (X, ρ) be a metric space . A mapping A: X → X is said to be sequentially convergent if we have ,for every sequence $\{x_n\}$, if $\{Ax_n\}$ is convergence, then $\{x_n\}$ is also convergence. A is said to be sub sequentially convergent if we have , for every sequence $\{x_n\}$, if $\{Ax_n\}$ is convergence then, $\{x_n\}$ has a convergent sub sequence.

Definition 1.2. Let X be a nonempty set . Suppose that the mapping A:X→X , satisfies

(i) ρ(x, y) ≥ 0 for all x, yϵX and ρ(x, y) = 0 if and only if x = y;

(ii) ρ(x, y) = ρ(y, x) for all x, yϵX ;

(iii) ρ(x, y) ≤ ρ(x, w) + ρ(w, z) + ρ(z,y) for all x, yϵX and for all distinct points w,z ϵ X\{x, y}. Then d is called a generalized metric and (X, d) is a generalized metric space.

3. MAIN RESULTS

Theorem 2.1. Let (X, ρ) be a CM-Space and A, B :X→X be mappings such that A is continuous, one-to-one and sub sequentially convergent . If αϵ[0. ½) and

$$\rho(ABx, ABy) \leq \alpha [\rho(ABx, ABy) + \rho(ABx, ABy)]. \quad \ldots (1)$$

For all x,y ϵX, then b has a unique fixed point . Also A is a sequentially convergent , then for every x_0ϵX the sequence of iterates $\{B^n x_0\}$ converges to this fixed point.

Proof: Let x_0 be an arbitrary point in X . We define the iterative sequence $\{x_n\}$ by $x_{n+1} = Bx_n$ (equivalently $x_n = B^n x_0$) n = 1,2,3.. using (1) we have

$$\rho(Ax_n, Ax_{n+1}) = \rho(ABx_{n-1}, ABx_n)$$
$$\leq \alpha[\rho(Ax_{n-1}, ABx_n) + \rho(Ax_n, ABx_{n-1})]$$
$$\leq \alpha[\rho(Ax_{n-1}, Ax_{n+1}) + \rho(Ax_n, Ax_n)]$$
$$\leq \alpha[\rho(Ax_{n-1}, Ax_n) + \rho(Ax_n, Ax_{n+1})]$$
$$\rho(Ax_n, Ax_{n+1}) \leq \alpha/1-\alpha\,[\rho(Ax_{n-1}, Ax_n)]$$
$$\leq h\ \rho(Ax_{n-1}, Ax_n)\ \text{, where } h = \alpha/1-\alpha < 1. \quad \ldots (2)$$

By the same argument

$$\rho(Ax_n, Ax_{n+1}) \leq h\,\rho(Ax_{n-1}, Ax_n) \leq h^2\rho(Ax_{n-2}, Ax_{n-1}) \leq \ldots \leq h^n\rho(Ax_0, Ax_1)\ . \quad \ldots (3)$$

By (3) for every m,n $\epsilon\mathbb{N}$ such that m > n we have

$$\rho(Ax_m, Ax_n) \leq \rho(Ax_m, Ax_{m-1}) + \rho(Ax_{m-1}, Ax_{m-2}) + \ldots + \rho(Ax_{n+1}, Ax_n)$$
$$\leq [h^{m-1} + h^{m-2} + \ldots + h^n]\,\rho(Ax_0, Ax_1)$$
$$= h^n\,[1 + h + h^2 + \ldots]\,\rho(Ax_0, Ax_1)$$
$$\leq h^n / 1-h\ \rho(Ax_0, Ax_1)$$, letting m, n $\to\infty$ we get that

$\{Ax_n\}$ is a Cauchy sequence and X is a CM-Space there exists qϵX such that $\lim_{n\to\infty} Ax_n = q$. … (4)

Since A is sub sequentially convergent , $\{x_n\}$ has has a sub sequence . So there exists pϵX and $\{x_{n(r)}\}$ (r = 1, 2, .. ∞) such that $\lim_{r\to\infty} x_{n(r)} = p$. … (5)

Since A is continuous and $\lim_{r\to\infty} x_{n(r)} = p$, $\lim_{r\to\infty} Ax_{n(r)} = Ap$. By (5) we conclude that Ap= q. So

$$\rho(ABp, Ap) \leq \rho(ABp, AB^{n(r)}x_0) + \rho(AB^{n(r)}x_0, AB^{n(r)+1}x_0) + \rho(AB^{n(r)+1}x_0, Ap)$$
$$\leq \alpha\,[\rho(ABp, AB^{n(r)}x_0) + \rho(AB^{n(r)-1}x_0, ABp)] + h^{n(r)}\rho(Ax_1, Ax_0) + \rho(Ax_{n(r)+1}, Ap)$$
$$\leq \alpha\,[\rho(ABp, Ax_{n(r)}) + \rho(Ax_{n(r)-1}, ABp)] + h^{n(r)}\rho(Ax_1, Ax_0) + \rho(Ax_{n(r)+1}, Ap).$$ Letting n(r) $\to\infty$
$$\leq \alpha\,[\rho(ABp, Ap) + \rho(Ap, ABp)]$$
$$\leq 2\alpha[\rho(ABp, Ap)]$$

$\rho(ABp, Ap) \leq 0$. Implies that ABp = Ap .Since A is one –to –one we get that Bp= p. Therefore B has a fixed point. Since (1) holds and A is one –to-one, B has a unique fixed point.

Now if A is sequentially convergent by replacing {n} with {n(r)} we conclude that $\lim_{r\to\infty} x_n = p$ and this shows that $\{x_n\}$ converges to the fixed point of B.

This completes the proof of the theorem.

Conclusion: Our results are general and they are the generalization results of [7].

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