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FaceandTotalFaceProductCordialLabelinginSomeGraphFamilies

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Abstract

Foraplanargraph$G$,thevertexlabelingfunctionisdefinedas$g$: $V(G)\rightarrow\{0,1\}$and$g(v)$iscalledthelabelofthevertex$v$of$G$under$g$,inducededgelabelingfunctiong^{*}:$E(G)\rightarrow\{0,1\}$isgivenasife $=uv$then $g^{*}(uv) = g(u)g(v)$ and induced face labeling function g^{**}: $F(G) \rightarrow \{0, 1\}$ is given as if $v_1, v_2, \dots, v_n$ande$_1$, e$_2$, ... , e$_m$are the vertices and edges of face f then $g^{**}(f) = g(v_1)g(v_2) \dots g(v_n)g^{*}(e_1)g^{*}(e_2) \dots g^{*}(e_m)$.Letusdenote$v_g(i)$ isthenumberofverticesof$G$havinglabeliunder$g$,e$_g$(i)isthenumberofedgesofGhavinglabeliunder$g^{*}$and$F_g(i)$isthenumberofinteriorfaces of Ghavinglabeliunderg^{**},for i∈{0,1}.$g$iscalled thefaceproductcordiallabelingofgraphGif $|v_g(0) - v_g(1)|\leq 1$, $|e_g(0) - e_g(1)|\leq 1akd|F_g(0) - F_g(1)|\leq 1$.Agraph$G$isafaceproductcordialgraphifitadmitsfaceproductcordiallabeling.Let$g(0)$and $g(1)$bethesumofthenumberofvertices,edgesandinteriorfaceshavinglabels0 andlabels1respectively.$g$iscalled total face product cordial labeling of graph Gif $|g(0) - g(1)| \leq 1$. A graph $G$is called total face productcordialgraphifitadmitstotalfaceproductcordiallabeling.Faceproductcordiallabelingisinvestigatedforbookgraph when $k$is odd and $m\leq 2$. Face product cordial and total face product cordial labeling of gear graph isexploredwhen$k\geq 3$.Also,thegraphobtainedfromageargraphbyduplicatingeachvertexofdegreetwobyanedge, the graph obtained from a gear graph by duplicating each vertex of degree three by an edge and the graphobtained by duplication of all rim vertices of a gear graph by an edge when $k\leq 3$ is face product cordial andtotalfaceproduct cordialgraph.

Keywords: BookGraph,Duplication,Faceproductcordiallabeling, Geargraph,Productcordiallabeling,Totalfaceproduct cordial labeling.

AMSSubjectClassification(2020):05C78,05C76.

1. Introduction

Agraphlabelingistheassignmentofintegerstotheverticesoredgesorbothsubjecttoconditions. FordifferentgraphlabelingtechniquesweuseadynamicsurveyofgraphlabelingbyGallian[1].Westartwithasimple,finite,undirectedgraph$G=(V(G),E(G))$withvertexset$V$andedgeset$E$of$G$.Fordifferentnotationandterminology we follow Gross and Yellen [2]. Face Cordial labeling and Total face product cordial labeling weregivenbyP.LawrenceRozarioRajandR.LawrenceJosephManoharan[3].Nowweprovidebriefsummaryofdefinitionsandother information which are necessary forthepresentinvestigations.

2. Definitions

Definition2.1 Acycleinagraphisanon-emptytrailinwhichonlythefirstandlastverticesareequal.

Definition2.2 ThegraphW$_n$=C$_n$+K$_1$iscalledawheelgraph.ThevertexcorrespondingtoK$_1$iscalledtheapex vertexandthe verticescorrespondingtoCnarecalledthe rim vertices[1].

Definition2.3 AbookgraphB(m,n)isagraphobtainedbyidentifyingnedgestakingoneedgefromeachofthendistinctcopiesof C$_m$,whereniscallednumberofpagesofthebookB(m,n).Theedgeobtainedbyidentifyingnedgesiscalledthespineor baseof thegraphB(m,n)[4].

Definition 2.4 A gear graph G_n is obtained from the wheel graph $W_n$by adding a vertex between every pair ofadjacentverticesin thecycleC_n[5].

Definition 2.5A mapping f:V (G)→{0, 1}is calledabinary vertexlabeling of Gand f(v)is calledthe labelof the vertex v of G under f. We denote $v_f(0)$ as the number of vertices with label 0 and $v_f(1)$ as the number ofverticeswithlabel1[5].

Definition 2.6 A product cordial labeling of graph G with vertex set V is a function f : V (G) → {0, 1} such thateach edge uv is assigned the label f(u)f(v), the number of vertices with label 0 and the number of vertices withlabel 1 differ by atmost 1 and the number of edge with label 0 and the number of edge with label 1 differ byatmost1.Agraphwhichadmitsproductcordiallabelingiscalledaproductcordialgraph.Sundaram,PonrajandSomasundaram[6]introducedaproductcordiallabeling.

Definition2.7 Theneighbourhoodof avertexvofagraph isthesetof allverticesadjacenttov.Itis denotedby N(v)[5].

Definition2.8 DuplicationofavertexofthegraphGisthegraphG′obtainedfrom Gbyadding anewvertexv′ tosuchthatN(v′)= N(v)[5].

Definition 2.9 Duplication of a vertex $v_k$by a new edge e=v_k,v'_k in a graph G produces a new graph G′ suchthat N(v'_k)={v_k, v''_k} and N(v''_k)={v_k, v'_k}. The symbols of duplication of a vertex by a new edge andduplicationof an edgebyanewvertex werefoundbyVaidyaandBansara[7].

Definition 2.10 For a planar graph G, the binary vertex labelingfunctionis definedas g:V (G)→{0, 1}andg(v)is the label of the vertex v of Gunder g, induced edge labeling function g^*:E(G)→{0, 1} is given as if e =uv then $g^*(uv)$= g(u)g(v) and induced face labeling function g^{**}: F(G) → {0, 1} is given as if v_1, v_2, … , $v_n$and e_1, e_2, … , $e_m$are the vertices and edges of face f then $g^{**}(f) = g(v_1)g(v_2) . . . g(v_n)g^*(e_1)g^*(e_2) . . . g^*(e_m)$.Letusdenote$v_g(i)$isthenumberofverticesofGhaving labeliunderg,$e_g(i)$isthenumberofedgesofGhavinglabeliunder$g^*$and$f_g(i)$isthenumberofinteriorfacesofGhavinglabeliunderg^{**}fori=1,2.giscalledthe faceproductcordiallabelingofgraph G if if$|v_g(0) - v_g(1)| \leq 1$, $|e_g(0) - e_g(1)| \leq 1$ and $|F_g(0) - F_g(1)| \leq 1$. A graph G is a face product cordial graph if it admits face product cordial labeling [3]. P. LawrenceRozarioRaj andR.LawrenceJosephManoharan [3]introducedafaceproductcordiallabeling.

Definition 2.11 For a planar graph G, the binary vertex labelingfunctionis definedas g:V (G)→{0, 1}andg(v)is the label of the vertex vof G under g, induced edge labeling function g^*:E(G)→{0, 1}is given as if e=uv then $g^*(uv)$=g(u)g(v) and induced face labeling function g^{**} :F(G)→ {0, 1} is given as if v_1, v_2, … , $v_n$ande_1,e_2,…,$e_m$aretheverticesandedgesoffacesftheng**(f)=$g(v_1)g(v_2)…g(v_n)g^*(e_1)g^*(e_2)…g^*(e_m)$.Let g(0) and g(1) be the sum of the number of vertices, edges and interior faces having labels 0 and labels 1respectively. g is called total face product cordial labeling of graph G if $|g(0) - g(1)| \leq 1$. A graph G is calledtotalfaceproductcordialgraphifitadmitstotalfaceproductcordiallabeling[3].P.LawrenceRozarioRajand R.LawrenceJosephManoharan[3]introducedatotalfaceproductcordiallabeling.

Note:-Throughoutthe articleweusei∈ $[k]$,whenever$1 \leq i \leq k$.

3. Results

Theorem*3.1*Abookgraph$B(m,k)$isfaceproductcordialgraphifboth$m$and$k$areoddand$m$≥3.

Proof: Let $B(m, k)$ be the book graph with both $m$and $k$are odd with m≥2 is obtained by identifying $k$edgestaking one edge from each of the $k$distinct copies of C_m, where $k$is called number of the pages of $B(m, k)$. Theedge obtained by identifying $k$edges is called the spine. Thus, $|V(B(m, k))| = (m - 2)k + 2$, $|E(B(m, k))| =(m-1)k+ 1akd|F(B(m,k))|=k$.Nametheverticesof$G$asfollows:$v_0andv'_0$totheendverticesofthe spinein$B(m,k)$.Letv_{11},v_{21},v_{31},…,$v_{(m-2)n}$betheconsecutive vertices of1stcopyofC_m,2ndcopyofC_m,…, k^{th}copyof$C_m$otherthan theendverticesofthespine.Define f:$V(B(m,k))$→{0,1}asfollows:

$$f(x)=\begin{cases}1, & \text{if } x\in\{v_0,v'_0\};\\ 1, & \text{if } x=v_{ij},\ i\in[\frac{m-1}{2}], j\in[\frac{k-1}{2}];\\ 1, & \text{if } x=v_{ij}, i=1, j\in[\frac{k+1}{2}];\\ 0, & otherwise.\end{cases}$$

Inviewofabovelabelingpatternwehave, $v_f(1)=\frac{(m-2)n+3}{2}$, $v_f(0)=\frac{(m-2)n+1}{2}$, $e_f(1)=\frac{(m-1)n}{2}$, $e_f(0)=\frac{(m-2)n+1}{2}$, $F_f(1)=\frac{n-1}{2}$ and $F_f(0)=\frac{n+1}{2}$. Thus, $|e_f(0)-e_f(1)|\le 1$, $|v_f(0)-v_f(1)|\le 1$ and $|F_f(0)-F_f(1)|\le 1$. Hence, B(m,n) isafaceproductcordialgraphwhenmandnbothoddwithm≥3asitadmitsface productcordiallabeling.

Illustration3.1:Faceproductcordiallabelingof$B(5,3)$isshownintheFigure1.

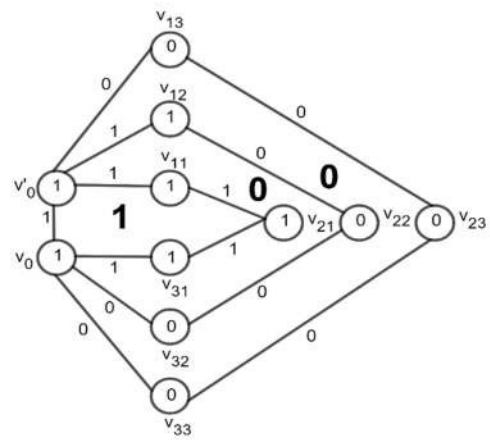

Figure1:Faceproductcordiallabelingof$B(5,3)$

Theorem*3.2* GeargraphGnisafaceproductcordialandtotalfaceproductcordialgraphforn≥3foroddvaluesofn.
Proof: Let$W_n$beawheelgraphwiththeapexvertex$v_0$andconsecutiverimverticesas$v_1,v_2,...,v_n$.Thegeargraph$G_n$isobtainedby subdividingeachoftherimedges$v_1v_2,v_2v_3,...,v_{n-1}v_n,v_nv_1ofW_n$bythevertices $u_1,u_2,...,u_n$respectively.Thus, $|V(G_n)|=2k+1$, $|E(G_n)|=3k$ and $|F(G_n)|=k$.

Defineafunction $f:V(G)\to\{0,1\}$asfollows:

$$f(x)=\begin{cases}1, & \text{if } x=v_0;\\ 1, & \text{if } x=u_i, i\in[\frac{k-1}{2}];\\ 1, & \text{if } x=v_i, i\in[\frac{k+1}{2}];\\ 0, & otherwise.\end{cases}$$

Inviewofabovelabelingpatternwehave, $v_f(0)=k$, $v_f(1)=k+1$, $e_f(0)=\frac{3n+1}{2}$, $e_f(1)=\frac{3n-1}{2}$, $F_f(0)=\frac{n+1}{2}$, $F_f(1)=\frac{n-1}{2}$. Thus, $|e_f(0)-e_f(1)|\le 1$, $|v_f(0)-v_f(1)|\le 1$ and $|F_f(0)-F_f(1)|\le 1$. Hence, $G_n$isfaceproductcordialgraphforallodd$k$and$k\ge 3$. Also, $g_f(0)=v_f(0)+e_f(0)+F_f(0)=k+\frac{3n+1}{2}+\frac{n+1}{2}=3k+1$ and $g_f(1)=v_f(1)+e_f(1)+F_f(1)=(k+1)+\frac{3n-1}{2}+\frac{n-1}{2}=3k$. Thus, $|g_f(0)-g_f(1)|\le 1$. Hence,itistotalfaceproductcordialgraph.

Illustration3.2: FaceproductcordiallabelingandtotalfaceproductcordiallabelingofG_3isshownintheFigure2.

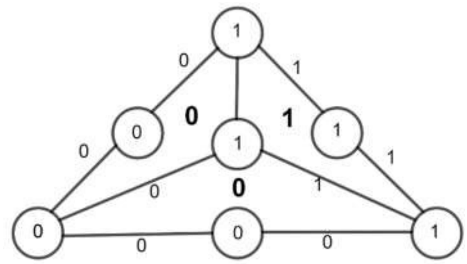

Figure2:FaceprodctcordiallabelingofG_3.

Theorem*3.3*Thegraphobtainedbyduplicationofeachvertexofdegreetwobyanedgein$G_n$admitsfaceproductcordial labeling and totalfaceproduct cordial labelingbothfor$k$≥3.

Proof: LetW$_n$beawheelgraphwiththeapexvertex$v_0$andconsecutiverimverticesasv_1,v_2,...,v_n.Thegeargraph$G_n$isobtainedby subdividingeachoftherimedgesv_1v_2,v_2v_3,...,$v_{n-1}v_n$,$v_nv_1$ofW$_n$bythevertices u_1, u_2, ... , $u_n$respectively. Thus, $|V(G_n)|=2k+1$, $|E(G_n)|=3kakd$ $|F(G_n)|=$ k. Let Gbe the graphobtainedfrom$G_n$byduplicatingeachvertexu_i ofdegreetwobyanedge$u'_iu''_i$ respectivelyforalli =1,2,3,...,k.Thus,$|V(G)|=4k+1$,$|E(G)|=6kakd|F(G)|=2k$.

Defineafunction f:$V(G) \rightarrow \{0,1\}$asfollows:

$$f(x)=\begin{cases}1, & \text{if} x=v_0;\\ 1, & \text{if} x=v_i, i\in[k];\\ 1, & \text{if } x=u_i, i\in[k];\\ 0, & \text{if} x\in\{u'_iu''_i\}, i\in[k].\end{cases}$$

Inview of above labeling pattern we have, $v_f(0)=2k$,$v_f(1)=2k+1$,$e_f(0)=3k$,$e_f(1)=3k$,$F_f(0)=k$, $F_f(1)=k$. Thus, $|e_f(0)-e_f(1)| \leq 1$, $|v_f(0) - v_f(1)| \leq 1\ akd\ |F_f(0) - F_f(1)| \leq 1$. Hence, $G_n$isface product cordial graph for all odd $k$and $k\geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 2k + 3k + k = 6k$and $g(1) = v_f(1) + e_f(1) + F_f(1) = (2k + 1) + 3k + k = 6k + 1$. Thus, $|g(0)-g(1)| \leq 1$. Hence, it istotalfaceproductcordialgraph.

Illustration3.3:FaceproductcordiallabelingandtotalfaceproductcordiallabelingofG_5isshownintheFigure 3.

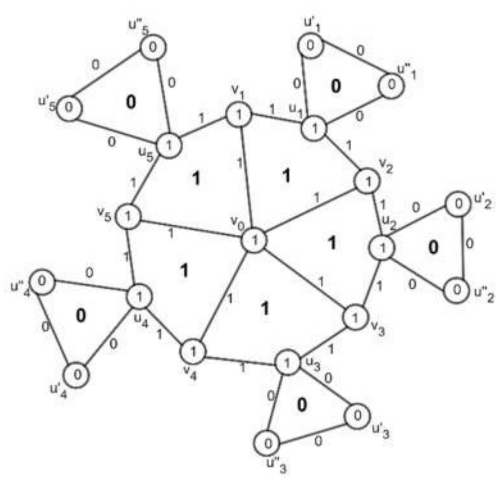

Figure3:FaceprodctcordiallabelingofG_5.

Theorem*3.4*Thegraphobtainedbyduplicationofeachvertexofdegreethreebyanedgein$G_n$isbothfaceproductcordial graphand totalfaceproduct cordialgraph.

Proof: Let $W_n$be a wheel graph with the apex vertex $v_0$and consecutive rim vertices as v_1,v_2, ... , v_n. The geargraph$G_n$isobtainedbysubdividingeachoftherimedgesv_1v_2,v_2v_3,...,$v_{n-1}v_n$,$v_nv_1$of$W_n$bythevertices u_1, u_2, ... , $u_n$respectively. Thus, $|V(G_n)|=2k+1$, $|E(G_n)|=3k$akd $|F(G_n)|= k$. Let Gbe the graphobtained from $G_n$by duplicating each vertex $v_i$of degree three by an edge $v'_iv''_i$respectively for all i =1,2,3,...,k.Thus,$|V(G)|=4k+ 1$,$|E(G)|=6k$akd$|F(G)|=2k$.

Defineafunction f:$V(G) \rightarrow \{0,1\}$asfollows:

$$f(x)=\begin{cases}1, & \text{if } x=v_0;\\ 1, & \text{if } x=v_i, i\in[k];\\ 1, & \text{if } x=u_i, i\in[k];\\ 0, & \text{if } x\in\{v'_iv''_i\}, i\in[k].\end{cases}$$

Inview of above labeling pattern we have, $v_f(0)=2k$,$v_f(1)=2k+1$,$e_f(0)=3k$,$e_f(1)=3k$,$F_f(0)=k$, $F_f(1)=k$. Thus, $|e_f(0) - e_f(1)| \leq 1$, $|v_f(0) - v_f(1)| \leq 1$ akd $|F_f(0) - F_f(1)| \leq 1$. Hence, $G_n$isface product cordial graph for all odd $k$and $k\geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 2k + 3k + k = 6k$and $g(1) = v_f(1) + e_f(1) + F_f(1) = (2k + 1) + 3k + k = 6k + 1$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it istotalfaceproductcordialgraph.

Illustration3.4: FaceproductcordiallabelingandtotalfaceproductcordiallabelingofG_4isshownintheFigure 4.

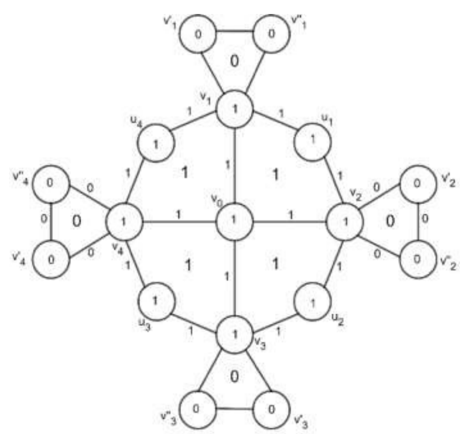

Figure4:FaceprodctcordiallabelingofG_4

Theorem*3.5*Thegraphobtainbyduplicationofeachoftherimverticesin$G_n$isbothfaceproductcordialgraphandtotalfaceproductcordialgraphif$k$iseven.

Proof: Let$W_n$beawheelgraphwiththeapexvertex$v_0$andconsecutiverimverticesasv_1,v_2,...,v_n.Thegeargraph$G_n$isobtainedby subdividingeachoftherimedgesv_1v_2,v_2v_3,...,$v_{n-1}v_n$,$v_nv_1$of$W_n$bythevertices u_1, u_2, ... , $u_n$respectively. Thus, $|V(G_n)|=2k+1$, $|E(G_n)|=3k$akd $|F(G_n)|= k$. Let Gbe the graphobtained from $G_n$by duplicating each vertex $v_i$and $u_i$by an edge $v'_iv''_i$and $u'_iu''_i$ respectively for all i =1,2,3,...,k.Thus,$|V(G)|=6k+ 1$,$|E(G)|=9k$akd$|F(G)|=3k$.

Case1:n≡0(mod4).

Defineafunction f:$V(G) \rightarrow \{0,1\}$asfollows:

$$f(x)=\begin{cases}1, & \text{if } x=v_i, i\in\{0,[k]\};\\ 1, & \text{if } x=u_i, i\in[k];\\ 1, & \text{if } x=v'_i, i\in[\frac{k}{4}];\\ 1, & \text{if } x=u'_i, i\in[\frac{k}{4}];\\ 1, & \text{if } x=u''_i, i\in[\frac{k}{4}];\\ 0, & otherwise.\end{cases}$$

Inviewofabovelabelingpatternwehave, $v_f(0)=3k, v_f(1)=3k+1, e_f(0)=\lceil\frac{9n}{2}\rceil, e_f(1)=\lfloor\frac{9n}{2}\rfloor, F_f(0)=\lceil\frac{3n}{2}\rceil, F_f(1)=\lfloor\frac{3n}{2}\rfloor$. Thus, $|e_f(0)-e_f(1)|\leq 1, |v_f(0)-v_f(1)|\leq 1$ and $|F_f(0)-F_f(1)|\leq 1$. Hence, G isfaceproductcordialgraphforalloddkand $k\geq 3$. Also, $g(0)=v_f(0)+e_f(0)+F_f(0)=3k+\lceil\frac{9n}{2}\rceil+\lceil\frac{3n}{2}\rceil=9k$ and $g(1)=v_f(1)+e_f(1)+F_f(1)=(3k+1)+\lfloor\frac{9n}{2}\rfloor+\lfloor\frac{3n}{2}\rfloor=9k+1$. Thus, $|g(0)-g(1)|\leq 1$. Hence, itistotalfaceproductcordialgraph.

Case2: $n\equiv 1 \pmod 4$.
Defineafunction $f: V(G)\rightarrow\{0,1\}$ suchthat,

$$f(x)=\begin{cases}1, & \text{if } x=v_i, i\in\{0,[k]\};\\ 1, & \text{if } x=u_i, i\in[k];\\ 1, & \text{if } x=v'_i, i\in[\frac{k+3}{4}];\\ 1, & \text{if } x=v''_i, i\in[\frac{k-1}{4}];\\ 1, & \text{if } x=u'_i, i\in[\frac{k-1}{4}];\\ 1, & \text{if } x=u''_i, i\in[\frac{k-1}{4}];\\ 0, & otherwise.\end{cases}$$

In view of above labeling pattern we have, $v_f(0)=3k, v_f(1)=3k+1, e_f(0)=\lceil\frac{9n+1}{2}\rceil, e_f(1)=\lfloor\frac{9n-1}{2}\rfloor, F_f(0)=\lceil\frac{3n+1}{2}\rceil, F_f(1)=\lfloor\frac{3n-1}{2}\rfloor$. Thus, $|e_f(0)-e_f(1)|\leq 1, |v_f(0)-v_f(1)|\leq 1$ and $|F_f(0)-F_f(1)|\leq 1$. Hence, G isfaceproductcordialgraphforalloddkand $k\geq 3$. Also, $g(0)=v_f(0)+e_f(0)+F_f(0)=3k+\lceil\frac{9n+1}{2}\rceil+\lceil\frac{3n+1}{2}\rceil=9k+1$ and $g(1)=v_f(1)+e_f(1)+F_f(1)=(3k+1)+\lfloor\frac{9n-1}{2}\rfloor+\lfloor\frac{3n-1}{2}\rfloor=9k$. Thus, $|g(0)-g(1)|\leq 1$. Hence, itistotalfaceproductcordialgraph.

Case3: $n\equiv 2 \pmod 4$.
Defineafunction $f: V(G)\rightarrow\{0,1\}$ suchthat,

$$f(x)=\begin{cases}1, & \text{if } x=v_i, i\in\{0,[k]\};\\ 1, & \text{if } x=u_i, i\in[k];\\ 1, & \text{if } x=v'_i, i\in[\frac{k+2}{4}];\\ 1, & \text{if } x=v''_i, i\in[\frac{k+2}{4}];\\ 1, & \text{if } x=u'_i, i\in[\frac{k-2}{4}];\\ 1, & \text{if } x=u''_i, i\in[\frac{k-2}{4}];\\ 0, & otherwise.\end{cases}$$

Inviewofabovelabelingpatternwehave, $v_f(0)=3k, v_f(1)=3k+1, e_f(0)=\lceil\frac{9n}{2}\rceil, e_f(1)=\lfloor\frac{9n}{2}\rfloor, F_f(0)=\lceil\frac{3n}{2}\rceil, F_f(1)=\lfloor\frac{3n}{2}\rfloor$. Thus, $|e_f(0)-e_f(1)|\leq 1, |v_f(0)-v_f(1)|\leq 1$ and $|F_f(0)-F_f(1)|\leq 1$. Hence, G isfaceproductcordialgraphforalloddkand $k\geq 3$. Also, $g(0)=v_f(0)+e_f(0)+F_f(0)=3k+\lceil\frac{9n}{2}\rceil+\lceil\frac{3n}{2}\rceil=9k$ and $g(1)=v_f(1)+e_f(1)+F_f(1)=(3k+1)+\lfloor\frac{9n}{2}\rfloor+\lfloor\frac{3n}{2}\rfloor=9k+1$. Thus, $|g(0)-g(1)|\leq 1$. Hence, itistotalfaceproductcordialgraph.

Case4: $n\equiv 3 \pmod 4$.
Defineafunction $f: V(G)\rightarrow\{0,1\}$ suchthat,

$$f(x)=\begin{cases}1, & \text{if } x=v_i, i\in\{0,[k]\};\\ 1, & \text{if } x=u_i, i\in[k];\\ 1, & \text{if } x=v'_i, i\in[\frac{k+1}{4}];\\ 1, & \text{if } x=v''_i, i\in[\frac{k+1}{4}];\\ 1, & \text{if } x=u'_i, i\in[\frac{k+1}{4}];\\ 1, & \text{if } x=u''_i, i\in[\frac{k-3}{4}];\\ 0, & otherwise.\end{cases}$$

In view of above labeling pattern we have, $v_f(0)=3k, v_f(1)=3k+1, e_f(0)=\lceil\frac{9n+1}{2}\rceil, e_f(1)=\lfloor\frac{9n-1}{2}\rfloor, F_f(0)=\lceil\frac{3n+1}{2}\rceil, F_f(1)=\lfloor\frac{3n-1}{2}\rfloor$. Thus, $|e_f(0)-e_f(1)|\leq 1, |v_f(0)-v_f(1)|\leq 1$ and $|F_f(0)-F_f(1)|\leq 1$. Hence, G_n isfaceproductcordialgraphforallodd$k$and$k\geq 3$. Also, $g(0)=v_f(0)+e_f(0)+F_f(0)=3k+\lceil\frac{9n+1}{2}\rceil+\lceil\frac{3n+1}{2}\rceil=9k+1$ and $g(1)=v_f(1)+e_f(1)+F_f(1)=(3k+1)+\lfloor\frac{9n-1}{2}\rfloor+\lfloor\frac{3n-1}{2}\rfloor=9k$. Thus, $|g(0)-g(1)|\leq 1$. Hence,itistotalfaceproductcordialgraph.

Thus, by all the above cases it satisfies face and total face product cordial labeling if n is even. Hence the graphobtainbyduplication ofeachrimverticesin$G_n$admitsboth faceandtotalfaceproductcordiallabeling.

Illustration3.4: FaceproductcordiallabelingandtotalfaceproductcordiallabelingofG_8isshownintheFigure5.

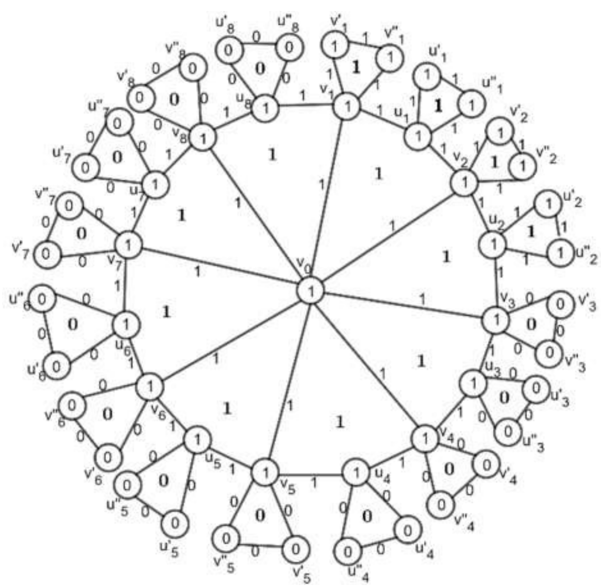

Figure5:FaceproductcordiallabelingG_8

4.Conclusion

We applied Face product cordial labeling on Book graph when m and n both odd with $m\geq 3$. Also we appliedFace product cordial labeling and Total Face product cordial labeling on Gear graph ($k\geq 3$). We also showedthreeresultsonthegraphobtainedbyswitchingofavertexofdistinctdegreesinthegeargraphisFaceandTotalface cordial labelingunderthecondition saiderlier.

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