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Ig*-CLOSEDSETSINFUZZYIDEALTOPOLOGICALSPACES

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Abstract:
Inthispaperweintroducethenotionofl g*–closedsets,Ig*-opensetsinfuzzyidealtopologicalspaceandstudiedsomeofitsbasicpropertiesandcharacterizations.
Itshowsthisclassliesbetweenfuzzyclosedsetsandfuzzyg–closedsets.

KeywordsandPhrases: *Ig*–closedsets,Ig*-open.*

1. Introduction

After the introduction of fuzzy sets by Zadeh [18] in 1965 and fuzzy topologybyChang [2]in1968, severalresearches were conducted on the generalization ofthe notions of fuzzy setsandfuzzytopology. The hybridization of fuzzy topologyand fuzzy ideal theory was initiated by Mahmoud [6] and Sarkar [12] independentlyin 1997. They [6, 12] introduced the concept of fuzzyideal topological spaces as anextensionoffuzzytopologicalspacesandidealtopologicalspaces.

Anonemptycollectionoffuzzy sets I of a set X satisfying theconditions:

(i) if$A\in I$ and$B\leq A$,then$B\in I$(heredity),

(ii) if $A\in I$ and $B\in I$ then $A\cup B\in I$(finite additivity)

iscalledafuzzy ideal on X. The triplex (X, τ, I) denotes a fuzzy idealtopologicalspacewithafuzzyidealIandfuzzytopologyτ[12].

Thelocal function for a fuzzy set A of X with respect to τ and Idenoted by $A^*(\tau,I)$ (brieflyA*)inafuzzy ideal topological space (X, τ,I) isthe union of all fuzzy points x_β such that if U is a Q-neighbourhood of $x_\beta$and $E\in I$ then for at least one point $y\in X$ for which $U(y) + A(y) - 1 > E(y)$ [12]. The ∗-closureoperator ofafuzzysetA denoted by $Cl^*(A)$ in (X,τ,I)defined as $Cl^*(A) = A \cup A^*$. In (X, τ, I) the collection $\tau^*(I)$ is anextensionoffuzzy topological space than τ via fuzzy ideal which isconstructed by considering the class $\beta = \{U-E : U \in \tau, E \in I\}$ as a base[6,12].

Recently the concepts of fuzzysemi-I-open sets [4], fuzzyα-I-open sets[16], fuzzy γ-I-open sets [3], fuzzy pre-I-open sets [8] and fuzzy δ-I-open sets [17]havebeen introduced and studied in fuzzy ideal topological spaces. In thepresentpaperweintroduceand study the concept of fuzzy I_{g*}-closed sets infuzzyidealtopologicalspaceswhichsimultaneouslygeneralizestheconceptofI_{g*} -closedsets[11].

2. Preliminaries

LetXbeanonemptyset. A family τ of fuzzy sets of X is calleda fuzzy topology [2]on X if the null fuzzy set 0andthewholefuzzyset1belongstoτandτisclosedwithrespecttoanyunionandfiniteintersection.Ifτisa fuzzy topology on X, then the pair (X, τ) is called a fuzzy topologicalspace.Themembersofτ are called fuzzy open sets of X and theircomplementsarecalledfuzzy closed sets. The closure of a fuzzy set A of Xdenoted by Cl(A), is the intersection of all fuzzy closed sets which contains A. Theinterior [2] ofafuzzysetAof X denoted by Int(A) is the union of all fuzzysubsetscontainedin A. A fuzzy set A of a fuzzy topological space (X, τ)is calledfuzzysemi-openifthereexistsafuzzyopensetUinXsuchthatU≤A≤Cl(U)[1].Afuzzy set A in (X, τ) is said to be quasi-coincident with a fuzzy setB, denoted by AqB, if there exists a point x ∈ X such that A(x) + B(x) > 1[4]. A fuzzy set V in (X, τ) is called a Q-neighbourhood of a fuzzypoint x_β if there exists a fuzzy open set U of X such that x_βqU ≤V[4].

Definition 2.1: A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzygeneralized closed written as fuzzy g-closed if Cl(A) ≤ O whenever A ≤ O and O isfuzzyopen[14].

Definition 2.2: A fuzzy set A of fuzzy ideal topological space (X, τ,I) is said to befuzzy∗-closed(resp.fuzzy∗-denseinitself)ifA^*≤A(resp.A≤A^*)[12].

Definition 2.3: Afuzzy set A of a fuzzy ideal topological space (X, τ, I) iscalled fuzzy I_g-closed if A^* ≤ U, whenever A ≤ U and U is fuzzy open in X[13].

Lemma2.1:A≤B⇔⌉(Aq(1−B)),foreverypairof fuzzy sets A and BofX[9].

3. **FuzzyIg*-closedsets**

Definition 3.1: A fuzzy set A of a fuzzy ideal topological space (X, τ, I) iscalledfuzzyIg*-closedifA^*≤U,wheneverA≤UandUisfuzzyg-openinX.

Remark 3.1: Every fuzzy ∗-closed set of a fuzzy ideal topological space (X, τ, I)isfuzzyIg*-closedandeveryfuzzyIg*-closedisfuzzyI_g-closedsetbuttheconversemaynotbetrue.

Remark 3.2: In a fuzzy ideal topological space (X,τ,I), I is fuzzy Ig*-closed foreveryA∈I.

Theorem 3.1: Let (X,τ, I)be a fuzzy ideal topological space. ThenA^* isfuzzyIg*-closedforeveryfuzzysetAofX.

Proof: LetAbea fuzzysetof XandUbeanyfuzzyg-opensetofXsuchthatA^*≤ U. Since $(A^*)^*$≤A^* it follows that $(A^*)^*$≤U. Hence A^* is fuzzy Ig*-closed.**Theorem3.2:**Let(X,τ,I)beafuzzyidealtopologicalspaceandAbeafuzzyIg*-closedandfuzzyg-opensetinX.ThenAisfuzzy∗-closed.

Proof: Since A is fuzzy g-open and fuzzy Ig*-closed and A ≤ A. It follows thatA^*≤A because A is fuzzy Ig*-closed. Hence Cl^*(A) = A∪A^*≤ A and A is fuzzy ∗-closed.

Theorem3.3: Let(X,τ,I)beafuzzyidealtopologicalspaceandAbeafuzzysetof X. Thenthefollowingareequivalent:

(i) AisfuzzyIg*-closed.

(ii) Cl^*(A)≤UwheneverA^*≤UandUisfuzzyg-openinX.

(iii) ⌉(AqF)⟹⌉(Cl^*(A)qF)foreveryfuzzyclosedsetFofX.

(iv) ⌉(AqF)⟹⌉(A^*qF)foreveryfuzzyclosedsetFofX.

Proof: (i)⟹(ii). Let A be a fuzzy Ig*-closed set in X. Let A^*≤ U where U is fuzzyg-open set in X. ThenA^*≤ U. Hence Cl^*(A) = A∪A^*≤ U. Which implies thatCl^*(A)≤U.

(ii)⟹(i). Let A be a fuzzy set of X. Byhypothesis Cl^*(A) ≤ U. Which implies thatA^*≤U.HenceAisfuzzy Ig*-closed.

(ii) ⟹(iii).LetFbeafuzzyclosedsetofXand ⌉(AqF).Then1−FisfuzzyopeninXandbyLemma2.1,A ≤1−F.Therefore,Cl^*(A) ≤1−F,becauseAisfuzzyIg*-closed.HencebyLemma2.1, ⌉(Cl^*(A)qF).

(iii) ⟹(ii).LetUbeafuzzyIg*-opensetofXsuchthatA^*≤U.ThenbyLemma2.1, ⌉(Aq(1−U)) and 1−U is fuzzy closed in X. Therefore by hypothesis ⌉(Cl*(A)q(1−U)).Hence,Cl^*(A)≤U.

(i)⟹(iv).LetFbeafuzzyg-closedsetinXsuchthat ⌉(AqF).ThenA≤1−Fwhere1−Fisfuzzyg-open.Therefore by(i)A^*≤1−F.Hence ⌉(A^*qF).

(iv)⟹(i).LetUbeafuzzyclosedsetinXsuchthatA≤U.ThenbyLemma2.1, ⌉(Aq(1−U)) and1−UisfuzzyclosedinX.Thereforebyhypothesis ⌉(A^*q(1−U)).HenceA^*≤UandAisfuzzyIg*-closedsetinX.

Theorem3.4: Let(X,τ,I)beafuzzyidealtopologicalspaceandAbeafuzzyIg*-closedset.ThenxqCl*(A)⟹Cl(x)qAforanyfuzzypointxofX.

Proof: LetxqCl*(A). If ⌉(Cl(x)qA). ThenbyLemma2.1,A≤(1−Cl(x)). And sobyTheorem3.3(ii),Cl^*(A)≤(1−Cl(x))because(1−Cl(x))isfuzzyg-opensetin X.WhichimpliesthatCl*(A)≤(1−x).HencebyTheorem3.3(ii), ⌉(xqCl*(A)),whichisacontradiction.

Theorem3.5: Let(X, τ, I) beafuzzyidealtopologicalspaceandAbefuzzy ∗-denseinitselffuzzyIg*-closedsetofX.ThenAisfuzzyg-closed.

Proof: Let U be a fuzzy open set of X such that A ≤ U. Since A is fuzzy Ig*-closed,byTheorem3.3(ii),Cl^*(A)≤U.Therefore,Cl(A)≤U,becauseAisfuzzy∗-denseinitself.HenceAisfuzzyg-closed.

Theorem3.6: Let(X,τ,I)beafuzzyidealtopologicalspacewhereI={0}andAbeafuzzysetofX.ThenAisfuzzyIg*-closedifandonlyifAisfuzzyg-closed.

Proof: SinceI={0},A^*=Cl(A)foreachsubsetAofX.Nowtheresultcanbeeasilyproved.

Theorem3.7: Let(X,τ,I)beafuzzyidealtopologicalspaceandA,BarefuzzyIg*-closedsetsofX.ThenAUBisfuzzy Ig*-closed.

Proof: LetUbeafuzzyg-opensetofXsuchthatAUB≤U.ThenA≤UandB≤U .ThereforeA^*≤UandB^*≤UbecauseAandBarefuzzyIg*-closedsetsofX.Hence$(AUB)^*$≤UandAUBisfuzzyIg*-closed.

Remark3.3: The intersection of two fuzzy Ig*-closed setsin afuzzyidealtopologicalspace(X,τ,I)maynotbefuzzyIg*-closed.

Example3.1: LetX={a,b}and A,Bbetwo fuzzysets defined as follows:

A(a)=0.9 , A(b)=0.7

B(a)=0.8 , B(b)=0.7

U(a)=0.3 , U(b)=0.4

Let τ ={ 0, U, 1} and I ={0}. Then A and B are fuzzyIg*-closedsetsin(X, τ, I) butA∩BisnotfuzzyIg*-closed.

Theorem 3.8: Let (X,τ,I) be a fuzzy ideal topological space and A, B are fuzzysetsofXsuchthatA≤B≤Cl^*(A).IfAisfuzzyIg*-closedsetinX,thenBisfuzzyIg*-closed.

Proof: Let U be a fuzzy g-open set such that B ≤ U.Since A ≤ B we have A ≤ U.Hence,Cl^*(A) ≤ U because A is fuzzy Ig*-closed.Now B ≤Cl^*(A) implies thatCl^*(B)≤Cl^*(A)≤U.ConsequentlyBisfuzzyIg*-closed.

Theorem 3.9: Let (X,τ,I) be a fuzzy ideal topological space and A, B are fuzzysetsofXsuchthatA≤B≤A^*.ThenAandBarefuzzyg-closed.

Proof: Obvious.

Theorem 3.10: Let(X,τ,I)be a fuzzy ideal topological space. If A and B arefuzzy subsets of X such that A ≤ B ≤ A^* and A is fuzzy Ig*-closed. ThenA^*=B^*andBisfuzzy∗-openinitself.

Proof: Obvious.

Theorem 3.11: Let(X,τ,I) be a fuzzy ideal topological space and $\mathcal{F}$ be thefamily of all fuzzy ∗- closed sets of X. Then τ ⊂ $\mathcal{F}$ if and only if everyfuzzysetofXisfuzzyIg*-closed.

Proof:Necessity. Let τ ⊂$\mathcal{F}$andU bea fuzzy g-opensetinXsuchthat A^*≤ U.Now U∈ τ⟹U ∈ $\mathcal{F}$. And so Cl^*(A)≤Cl^*(U)=U and A is fuzzyIg*-closedsetinX.

Sufficiency. Supposethatevery fuzzy set of X is fuzzy Ig*-closed.LetU∈τ.SinceUisfuzzyIg*-closedandU≤U,Cl^*(U)≤U.HenceCl^*(U) =UandU∈$\mathcal{F}$.Thereforeτ ⊂ $\mathcal{F}$.

Definition 3.2: A fuzzy set A of a fuzzy ideal topological space (X,τ,I) is calledfuzzyIg*-openifitscomplement1−AisfuzzyIg*-closed.

Remark 3.4: Every fuzzy ∗-open set in a fuzzy ideal topological space (X,τ, I) isfuzzyIg*-open and every fuzzyIg*-open is fuzzy I_g-open. But the converse maynotbetrue.

Theorem3.12: Let(X,τ,I)beafuzzyidealtopologicalspaceandAisfuzzysetof X. Then A is fuzzy Ig*-open if and only if F ≤ Int^*(A) whenever F is fuzzy g-closedandF≤A.

Proof:Necessity. LetAbefuzzyIg*-openandFisfuzzyg-closedsetsuchthatF≤ A. Then 1−A is fuzzy Ig*-closed, 1−A ≤ 1−F and 1− F is fuzzy g-open in X. HenceCl^*(1−A)≤(1−F).WhichimpliesthatF≤Int^*(A).

Sufficiency. LetU be afuzzyg-opensetsuch that1−A ≤ U.Then1−U isfuzzyg-closedsetofXsuchthat1−U ≤ A.Andsobyhypothesis,1−U ≤Int^*(A).Which implies that Cl^*(1−A) ≤ U and 1−A is fuzzy Ig*-closed. Hence A isfuzzyIg*-open.

Corollary3.1: Let(X,τ,I)beafuzzyideal topologicalspaceandAisfuzzysetof X. ThenAisfuzzyIg*-openifandonlyifF≤Int^*(A)wheneverFisfuzzyclosedandF≤ A.

Theorem3.13: Let(X,τ,I)beafuzzyidealtopologicalspaceandAbeafuzzysetofX.IfAisfuzzyIg*-open and Int^*(A)≤B≤A,then B is fuzzy Ig*-open.**Proof:**LetAbefuzzyIg*-openinXthen1−AisfuzzyIg*-closed.HenceCl^*(1−A) ≤(1−A)isfuzzyg-openset.AlsoInt^*(A)≤Int^*(B)⇒Cl^*(1−B)≤Cl^*(1−A).Hence,B is fuzzy Ig*-open.

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