Scopus

Journal of Vibration Engineering

ISSN:1004-4523

Registered

- SCOPUS
- DIGITAL OBJECT IDENTIFIER (DOI)
- GOOGLE SCHOLAR
- IMPACT FACTOR 6.1

Our Website
www.jove.science

GeneratingDio-3TriplesusingtheSecond-OrderPolynomialswithIncisiveProperties

JanakiG[1]andGowriShankariA[2]
[1]AssociateProfessor,CauveryCollegeforWomen(Autonomous),Trichy–18,India;
[2]AssistantProfessor,CauveryCollegeforWomen(Autonomous),Trichy–18,India;

Abstract: *Inthiscommunication,weachievespecialDiophantinetriplesinvolvingsecond-orderpolynomials,wheretheproductofanytwomembersofthesetsubtractedbytheir sum and increased by integer-coefficient polynomial yields a perfect square. AlsoprovidesgraphicalrepresentationoftheDio-3TriplesusingMATLAB.*

Keywords:SpecialDiophantineTriples,PerfectSquare,StarNumber.

1. INTRODUCTION

The enormous numbers of unresolved issues in number theory that appear to be solvablefrom the outside make it attractive. Unsolved issues in number theory are unsolved for areason,ofcourse.Althoughtheyappearto besimple,numbershave aremarkablycomplex structure that we only partially comprehend [9-13].Diophantus researched thefeaturethattheproductofanytwooftheirseparatecomponentsisonelessthanasquare hasaverylonghistory.If $x_i.x_j+n$isaperfectsquareforevery$1\leq i<j\leq s$,thena collectionof$s$uniquenon-nullintegers$(g_1,g_2,..............g_s)$isreferredtoasaDios-tuplewith attributes$D(n)$.AnumberofmathematiciansexploredtheexistenceofDiophantinetriples withtheproperty $D(n)$foranyinteger$n$and,moreover,foranylinearpolynomialin$n$. OnemightnowrecommendathoroughexaminationofmanytopicsrelatingtoDiophantinetriples[1–8].

Inthisstudy,weprovideuniqueDiophantinetriples (a,b,c) involvingpolynomials, where the product of any two members of the set, subtracted by their sum, and increasedbyaninteger-coefficientpolynomialisaperfectsquare.AlsoprovidesgraphicalrepresentationoftheDio-3TriplesusingMATLAB.

2. NOTATION

$star_n$:Starnumberofrank$n$=6n(n-1)+1

3. BASICDEFINITION

Asetofthreedifferentsecondorderpolynomialwithintegercoefficients $(a_1,a_2,a_3)$is saidtobeaspecialDiophantinetriplewithproperty $D(n)$ifa$_i*a_j-(a_i+a_j)+n$ isa perfectsquareforall$1\leq i<j\leq 3$,where$n$maybenon-zerointegerorpolynomialwithintegercoefficients.

4. ANALYTICALAPPROACH

4.1. Developmentofthedistinctivedio-3triplesusingthesecondorderpolynomial $6n^2-6n+1$ and $6n^2-18n+1$

Let $a=6n^2-6n+1$ and $b=6n^2-18n+1$

$$ab-(a+b)+72n+10=36n^4-144n^3+108n^2+72n+9$$
$$=(6n^2-12n-3)^2$$
$$=\lambda^2 \quad (1)$$

Equation(1)isaperfectsquare.

$ab-(a+b)+72n+10=\lambda^2$ where $\lambda=6n^2-12n-3$

Allowingctobeanon-zerointeger,

$$ac-(a+c)+72n+10=\mu^2 \quad (2)$$
$$bc-(b+c)+72n+10=\omega^2 \quad (3)$$

Solving(2)and(3)onemayget

$$(a-b)+(b-a)(72n+10)=(b-1)\mu^2-(a-1)\omega^2 \quad (4)$$

Setting $\mu=y+(a-1)T$ and $\omega=y+(b-1)T$

(5)

ApplyingEquation(5)in(4)onemayget

$y^2=(b-1)(a-1)T^2+72n+9$(6)

Initialsolutionof(6)isgivenby,

$y_0=(6n^2-12n-3)$ and $T_0=1$

Since $\mu=y+(a-1)T$ and $\omega=y+(b-1)T$,weobtainthat,

$\mu=12n^2-18n-3$

Therefore,theequation(2)becomes,

$$ac-c-a+72n+10=\mu^2$$
$$\Rightarrow(6n^2-6n)c=144n^4-432n^3+258n^2+30n$$
$$\Rightarrow c=24n^2-48n-5$$

Hence, the triples $(a,b,c)=(6n^2-6n+1, 6n^2-18n+1, 24n^2-48n-5)$ are Diophantinetripleswiththeproperty $D(72n+10)$.

Thefollowingtableprovidessomenumericalillustrations.

Table1

n	DiophantineTriples	$D(72n+10)$
1	(1,–11,–29)	82
2	(13,–**1**,–5)	154
3	(37,1,67)	226
4	(73,25,187)	298

RemarkableObservation:

It is noted that, the above second order polynomial is of theform $(a,b,c)=(star_n, star_{n-1}-12, 4star_{n-2}+72n-153)$ .Alsoallthetriplesareoddwithevennumberattributes.

4.2. Developmentofthedistinctivedio-3triplesusingthesecondorderpolynomial $6n^2-6n+1$ and $6n^2-30n+31$

Let $a=6n^2-6n+1$ and $b=6n^2-30n+31$

$$ab-(a+b)-36n+145=36n^4-288n^3+720n^2-576n+144$$
$$=(6n^2-24n+12)^2$$
$$=\lambda^2 \quad (7)$$

Equation(7)isaperfectsquare.

$ab-(a+b)-36n+145=\lambda^2$ where $\lambda=6n^2-24n+12$ Allowingc

tobeanon-zerointeger,

$$ac-(a+c)-36n+145=\mu^2 \quad (8)$$
$$bc-(b+c)-36n+145=\omega^2 \quad (9)$$

Solving(8)and(9)onemayget

$$(a-b)+(b-a)(-36n+145)=(b-1)\mu^2-(a-1)\omega^2 \quad (10)$$

Setting $\mu=y+(a-1)T$ and $\omega=y+(b-1)T$
(11)

ApplyingEquation(11)in(10)onemayget

$y^2=(b-1)(a-1)T^2-36n+145$(12)

Initialsolutionof(12)isgivenby,

$y_0=(6n^2-24n+12)$ and $T_0=1$

Since $\mu=y+(a-1)T$ and $\omega=y+(b-1)T$,weobtainthat,

$\mu=12n^2-42n+12$

Therefore,theequation(7)becomes,

$ac-c-a-36n+145=\mu^2$

$\Rightarrow(6n^2-18n)c=144n^4-1008n^3+2058n^2-990n$

$\Rightarrow c=24n^2-96n+55$

Hence, the triples $(a,b,c)=(6n^2-6n+1,6n^2-30n+31,24n^2-96n+55)$ are Diophantinetripleswiththeproperty $D(-36n+145)$.

Thefollowingtableprovidessomenumericalillustrations

Table2

n	**DiophantineTriples**	$D(-36n+145)$
1	(−11,7,−17)	109
2	(−11,−5,−41)	73
3	(1,−5,−17)	37
4	(25,7,55)	1

RemarkableObservation:

It is noted that, the above second order polynomial is of theform $(a,b,c)=(star_n, star_{n-2}-6, 4star_{n-3}+72n-237)$.Alsoallthetriplesandtheirattributesareodd.

4.3. Developmentofthedistinctivedio-3triplesusingthesecondorderpolynomial $6n^2-5$ **and** $6n^2-12n+13$

Let $a_n=6n^2-5$ and $a_{n-1}=6n^2-12n+13$

$$a_n a_{n-1}+30=36n^4-72n^3-24n^2+60n+25$$
$$=(6n^2-6n-5)^2$$
$$=\lambda^2 \quad (13)$$

Equation(7)isaperfectsquare.

$a_n a_{n-1} + 30 = \lambda^2$ where $\lambda = 6n^2 - 6n - 5$

Allowingctobeanon-zerointeger,

$a_n c + 30 = \mu^2$ (14)

$a_{n-1} c + 30 = \omega^2$ (15)

Solving(14)and(15)onemayget

$(a_n - a_{n-1})c = \mu^2 - \omega^2$
(16)

Setting $\mu = a_n + \lambda$ and $\omega = a_{n-1} + \lambda$
(17)

ApplyingEquation(17)in(16)onemayget

$c = a_n + a_{n-1} + 2\lambda$
$= 24n^2 - 24n - 14$ (18)

Hence, the triples $(a_n, a_{n-1}, c) = (6n^2 - 5, 6n^2 - 12n + 13, 24n^2 - 24n - 14)$ are Diophantinetripleswiththeproperty $D(30)$.

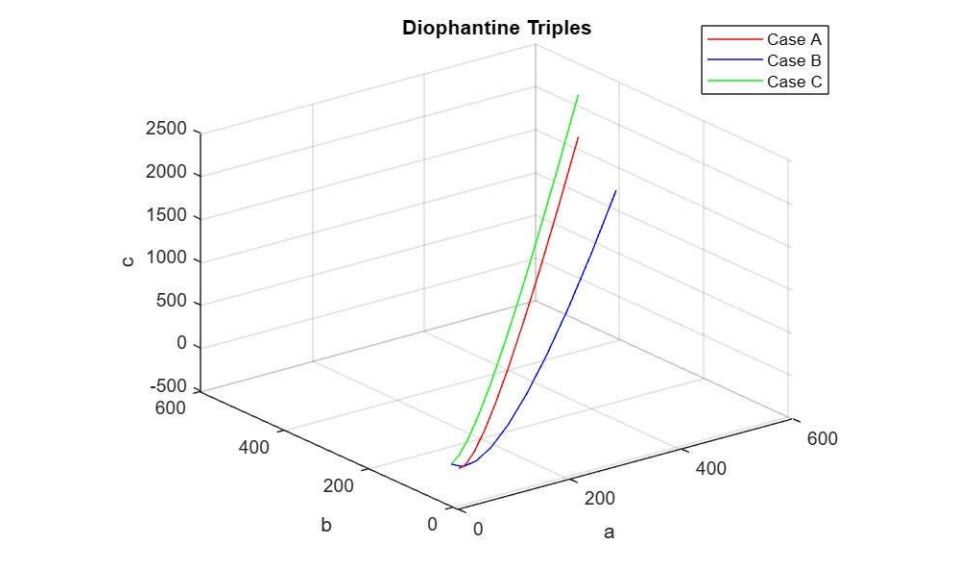
Thefollowingtableprovidessomenumericalillustrations

Table3

n	**DiophantineTriples**	$D(30)$
1	(1,–5,–14)	30
2	(19,1,34)	30
3	(49,19,130)	30
4	(91,49,274)	30

RemarkableObservation:

It is noted that, the above second order polynomial is of the form $(a_n, a_{n-1}, c) = (star_n + 6n - 6, star_{n-1} + 6n, 4star_{n-2} + 96n - 162)$



5. CONCLUSION

In this article, we have shown a few instances of how to build unique Dio 3 tuplesinvolving the second order polynomial with the right attributes. Also provides graphicalrepresentation of the Dio-3 Triples using MATLAB. In conclusion, one can look for Dio3tuplesforvariouspolynomials withtheircorrespondingattributes.

REFERENCES

[1] *Y.Fujita,"TheextendibilityofDiophantinepairs{k−1,k+1}",JournalofNumberTheory,128,* ***(2008)****,pp.322-353.[CrossRef]*

[2] *G.Janaki, and C.Saranya, "Special Dio 3-tuples for pentatope number", Journal of MathematicsandInformatics,vol.11,Specialissue,****(2017)****, pp.119-123.[CrossRef]*

[3] *G. Janaki, and C.Saranya, "Construction of the Diophantine Triple involving Pentatope Number",International Journal for Research in Applied Science & Engineering Technology, vol.6, no. III,****(2018)****,pp.2317-2319.[CrossRef]*

[4] *G.Janaki, and C.Saranya, "Half companion sequences of special dio 3-tuples involving Centeredsquare numbers", International Journal for Recent Technology and Engineering, vol.8, issue 3,****(2019)****,pp.3843-3845.[CrossRef]*

[5] *V.Pandichelvi and R.Vanaja, "Generating Diophantine Triples relating to figurate numbers withthought-provoking Property", Jnanabha, vol.52,no.2,****(2022)****, pp.106-110.*

[6] *C.Saranya, and G.Janaki, "Some Non-extendable Diophantine Triples involving Centered squarenumbers", International Journal of Scientific Research in Mathematical and Statistical Sciences,vol.6,no.6,****(2019)****,pp.105-107.*

[7] *C.Saranya, and G.Janaki , "Solution Of Exponential Diophantine Equation Involving JarasandhaNumbers", Advances and Applications in Mathematical Sciences, vol. 18, no. 12,* ***(2019)****, pp. 1625-1629.*

[8] *S.Vidhya,andG.Janaki,"ConstructionoftheDiophantine tripleinvolvingPronicnumber",International Journal for Research in Applied Science and Engineering Technology, vol. 6, no.1,****(2018)****,pp.2201-2204.*

[9] *R.D.Carmichael, "History of Theory of numbers and Diophantine Analysis", Dover Publication,Newyork,* ***(1959)****.*

[10] *L.J.Mordell,"Diophantineequations",Academicpress,London,****(1969)****.*

[**1**]*T.Nagell,"IntroductiontoNumbertheory",Chelseapublishingcompany,Newyork,****(1981)****.*

[12] *L.K.Hua,"IntroductiontotheTheoryofNumbers",Springer-Verlag,Berlin-Newyork,****(1982)****.*

[13] *OisteinOre,"NumbertheoryanditsHistory",Doverpublications,Newyork****(1988)****.*